Exercise 94

Use the Chain Rule and the Product Rule to give an alternative proof of the Quotient Rule. [*Hint:* Write $f(x)/g(x) = f(x)[g(x)]^{-1}$.]

Solution

Take the derivative of $f(x)/g(x)$.

$$\begin{aligned}
\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] &= \frac{d}{dx}\left\{f(x)[g(x)]^{-1}\right\} \\
&= \left\{\frac{d}{dx}[f(x)]\right\}[g(x)]^{-1} + f(x)\left\{\frac{d}{dx}[g(x)]^{-1}\right\} \\
&= f'(x)[g(x)]^{-1} + f(x)\left\{-[g(x)]^{-2}\cdot\frac{d}{dx}[g(x)]\right\} \\
&= f'(x)[g(x)]^{-1} + f(x)\left\{-[g(x)]^{-2}\cdot[g'(x)]\right\} \\
&= \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{[g(x)]^2} \\
&= \frac{f'(x)g(x)}{[g(x)]^2} - \frac{f(x)g'(x)}{[g(x)]^2} \\
&= \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}
\end{aligned}$$